# Determining tag orientation from acceleration and magnetic-field data using quaternions instead of Euler angles

## General points about quaternions

Quaternions are a way to encode rotations by using four numbers, the first one standing for the angle of the rotation and the three others for the coordinates of an axis vector.

In what follows, quaternions will be represented equivalently by 1x4 or 4x1 matrices.

A rotation of angle around the axis directed by the unit vector is represented bythe unit quaternion

A vector is represented in the quaternion space by.

The product of two quaternions is defined as

|  |  |
| --- | --- |
|  |  |

The inverse of a quaternion is : thus [1,0,0,0] which is the identity quaternion.

## Determining tag orientation: explanation of the Matlab code

The accelerometers measure the gravity field which is represented in the earth frame by the quaternion

The magnetometer measures the magnetic field which is represented in the earth frame by the quaternion

Let’s define the quaternion representing the rotation from the earth frame to the tag frame:

If there is no specific acceleration, the accelerometer gives the measurement of gravity in the tag frame:

And the magnetometer gives the measurement of magnetic field in the tag frame:

|  |  |
| --- | --- |
|  |  |
|  |  |

Let us define

This system will allow us to determine the unknowns. We need only 4 equations among those and we will use.

and can be determined as following :

being the scalar product of the vectors and.

can be then worked out from and :

However, we get some round-up errors by computing and then take its sine and cosine. To avoid part of these errors, we’ll replace and by and in and.

Knowing that, we can start solving the system **(I)**

(1)

(2)

(3)

(4)

(1) - (4) =>

(1) + (4) =>

(3) - (2) =>

(2) + (3) =>

From (1) - (4), we deduce that if, then or.

In that case, (3)-(2) allows us to decide between these two options:

If, then and.

If, then and.

If, then and.

If, then and.

We can then multiply (3) - (2) by on each side and replaceby, thus obtaining a quadratic equation in:

= 0. (5)

If, (3)-(2) gives us that which means that or.

Then with (1)-(4), and.

Else the discriminant of (5) is

The solutions are and

Whatever the sign of is, and; and we need a positive value as it is the value of.. We can see that this is also true in the case, though for the demonstration we had to tell it apart as there was a division by. Therefore it is not necessary to separate the two cases in the algorithm.

is simply obtained by (as we are in the case, ).

and are obtained in a very similar way using equations (1)+(4) and (2)+(3).

However system **(I)** is insufficient to determine entirely: if [] is a solution, [], [] and [] also are solutions of **(I)** but not all of them are solutions for our problem. A safe way to pick up a good one is to process and and take one that gives and. Note that and [] represent the same rotation, so the problem always has two solutions (except if the rotation is the identity rotation).